Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS P2

JUNE 2014 – COMMON TEST

MEMORANDUM

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

This memorandum consists of 11 pages.

QUESTION 1

1.1.1	$AC^2 = (-1 - 4)^2 + (-4 - 1)^2 = 9 + 9 = 45$ $AC = \sqrt{18}$ $= 3\sqrt{2}$ (3)	✓ correct substitution into distance formula ✓ $\sqrt{18}$ ✓answer
1.1.2	M is $\left(\frac{1-2}{2}; \frac{4-2}{2}\right) = \left(\frac{-1}{2}; 1\right)$ (2)	✓✓answer
1.1.3	Gradient of AB $= \frac{-2-4}{-2-1} = \frac{-6}{-3} = 2$ $\therefore$ gradient of $\perp$ bisector is $\frac{-1}{2}$ (obviously it passes through M) $\left(\frac{-1}{2}; 1\right)$ $y = -\frac{1}{2}x + c$ Substitute $1 = \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) + c$ $c = 1 - \frac{1}{4}$ $= \frac{3}{4}$ Equation is $y = -\frac{1}{2}x + \frac{3}{4}$ (4)	✓ $m_{AB} = 2$ ✓ $m_{\perp \text{bisector of AB}} = -\frac{1}{2}$ ✓ substitution of m and M into equation of line ✓answer
1.2	$x^2 - 2x + 1 + y^2 + 2y + 1 = 2x - 2y$ $x^2 - 4x + y^2 + 4y = -2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = -2 + 4 + 4$ $(x-2)^2 + (y+2)^2 = 6$ $\therefore$ centre is (2; –2) and radius $= \sqrt{6}$ (6) **[15]**	✓ $x^2 - 4x + 4$ ✓ $y^2 + 4y + 4$ ✓ $(x-2)^2$ ✓ $(y+2)^2$ ✓ answer: centre ✓ answer: radius

QUESTION 2

2.1.1	Since JK // LM, $M_{JK} = M_{LM}$ $\dfrac{-4-1}{p+2} = \dfrac{0-2}{5-3}$ $-2(p+2)=(-5)(2)$ $-2p-4=-10$ $-2p=-6$ $\therefore p=3$ (4)	✓equating gradients ✓✓substitution in each gradient ✓simplification
2.1.2	$JK^2 = (3+2)^2 + (-4-1)^2 = 50$ $LM^2 = (2-0)^2 + (3-5)^2 = 8$ $JK : LM = \sqrt{50} : \sqrt{8} = 5\sqrt{2} : 2\sqrt{2}$ $= 5 : 2$ (5)	✓ correct substitution into distance form. ✓ $\sqrt{50}$ ✓ $\sqrt{8}$ ✓ ratio ✓answer
2.1.3	Diagonals of a parallelogram bisect each other Midpoint of JL : $\left(\frac{3}{2};\frac{1}{2}\right)$ This is the same midpoint for MQ: Thus $\dfrac{x+3}{2} = \dfrac{3}{2}$; $\dfrac{y-2}{2} = -\dfrac{1}{2}$ $\therefore x = 0$ $\therefore y = -1$ Therefore Q is Q(0; –1) (5)	✓ $\left(\frac{3}{2};\frac{1}{2}\right)$ ✓ $\dfrac{x+3}{2} = \dfrac{3}{2}$ ✓ $\dfrac{y-2}{2} = -\dfrac{1}{2}$ ✓✓answer **OR** answer only(if done by inspection) – full marks
2.1.4	Since the *x* coordinates of K and M are both 3, it follows the equation of KM is $x = 3$. (2)	✓✓answer
2.1.5	90° since KM is a vertical line (1)	✓ answer
2.1.6	For collinearity; $m_{JR} = m_{JL}$ $m_{JR} = \dfrac{k-1}{1-(-2)} = \dfrac{0-1}{5-(2)}$ $= \dfrac{k-1}{3} = \dfrac{-1}{7}$ $\therefore k-1 = \dfrac{-3}{7}$ $\therefore k = \dfrac{-3}{7} + 1$ $k = \dfrac{4}{7}$ (4)	✓ equating: $m_{JR} = m_{JL}$ ✓ $m_{JR} = \dfrac{k-1}{1-(-2)}$ ✓simplification ✓answer

2.2.1	Q (x; 2) … (radius QR $\perp$ tangent) Substitute (x; 2) in $3x + 4y + 7 = 0$: $3x + 8 + 7 = 0$ $x = -5$ $\therefore$ Q (–5; 2) Radius = QR = 0 – (–5) = 5 $\therefore$ Equation is $(x + 5)^2 + (y - 2)^2 = 25$ (5)	✓ $y_Q = 2$ ✓substitution: $y = 2$ ✓ $x = -5$ ✓ radius ✓ equation
2.2.2	QR = 5 units d = 2 x radius $\therefore$ WZ = 10 units✓ (1) **[27]**	✓answer

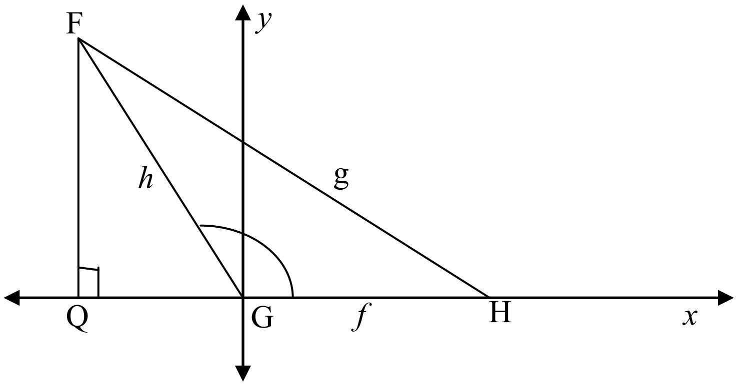
QUESTION 3

3.1	$\sin 15^\circ = \sin(45^\circ - 30^\circ)$ $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}} \ x \ \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$ (3)	✓15° as $45^\circ - 30^\circ$ ✓ correct expansion ✓ correct special angle values
	OR	
	$\sin 15^\circ = \cos 75^\circ$ $= \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \cos 30^\circ$ $= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}} \ \text{x} \ \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$ (3)	✓75° as $45^\circ + 30^\circ$ ✓ correct expansion ✓ correct special angle values
3.2	$\frac{\tan(180^\circ + \theta).\cos(360^\circ - \theta)}{\sin(180^\circ - \theta).\cos(90^\circ + \theta) + \cos(540^\circ + \theta).\cos(-\theta)}$ $\frac{\tan\theta.(\cos\theta)}{(\sin\theta).(-\sin\theta) - \cos\theta.\cos\theta}$ $= \frac{\frac{\sin\theta}{\cos\theta} \times \cos\theta}{-\sin^2\theta - \cos^2\theta}$ $= \frac{\sin\theta}{-(\sin^2\theta + \cos^2\theta)}$ $= -\sin\theta$ (9)	For each reduction : ✓ tan θ ✓ cos θ ✓ sin θ ✓– sin θ ✓– cos θ✓cos θ ✓ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓ $\sin^2\theta + \cos^2\theta$ ✓answer

3.3	$\sin(45^o + x).\sin(45^o - x) = \frac{1}{2}\cos 2x$ LHS $= \sin(45^o + x).\sin(45^o - x)$ $= (\sin 45^o \cos x + \cos 45^o \sin x)\sin 45^\circ \cos x - \sin x \cos 45^\circ)$ $= \left(\frac{1}{\sqrt{2}}.\cos x + \frac{1}{\sqrt{2}}.\sin x\right).\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$ $= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$ $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ $= \frac{1}{2}\cos 2x$ $=$ RHS (5)	✓✓ expansion ✓special ∠ values ✓ difference of squares ✓ HCF of $\frac{1}{2}$

3.4	$\frac{\sin 33^o}{\sin 11^o} - \frac{\cos 33^o}{\cos 11^o}$ $= \frac{\sin 33^o.\cos 11^o - \cos 33^o.\sin 11^o}{\sin 11^o.\cos 11^o}$ $= \frac{\sin(33^o - 11^o)}{\sin 11^o \cos 11^o}$ $= \frac{\sin 22^o}{\sin 11^o \cos 11^o}$ $= \frac{2\sin 11^o \cos 11^o}{\sin 11^o \cos 11^o}$ $= 2$ (6)	✓$\sin 33^o.\cos 11^o - \cos 33^o.\sin 11^o$ ✓$\sin 11^o.\cos 11^o$ ✓ $\sin(33^o - 11^o)$ ✓ $\sin 22^o$ ✓ $2\sin 11^o \cos 11^o$ ✓ answer
3.5	$\frac{\tan 3x}{\tan 24^o} = 1$ $\tan 3x = \tan 24^o$ $3x = 24^o + k.180^o$ $\therefore x = 8^o + k.60^\circ, k \in Z$ (5) **[28]**	✓ $\tan 24°$ ✓ $\tan 3x = \tan 24^o$ ✓ $3x = 24^o + k.180^o$ ✓ $\therefore x = 8^o$ ✓ $k.60^\circ, k \in Z$

QUESTION 4

<table>
<tr><td>4.1</td><td>

Constr: Draw FQ $\perp$ HG produced

$\sin\ F\hat{G}H = \sin(180° - F\hat{G}Q) = \sin\ F\hat{G}Q$

$\frac{FQ}{h} = \sin G$
$\therefore FQ = h \sin G$
Also $\frac{FQ}{g} = \sin H$
$\therefore FQ = g \sin H$
$\therefore h \sin G = g \sin H$
$\therefore \frac{\sin G}{g} = \frac{\sin H}{h}$
(5)</td><td>✓ construction

✓ reduction

✓ $\frac{FQ}{h} = \sin G$

✓ $\frac{FQ}{g} = \sin H$

✓ $h \sin G = g \sin H$</td></tr>
<tr><td>4.2.1</td><td>$\frac{PW}{PQ} = \tan\alpha$ ✓
$\therefore PW = PQ \tan\alpha$ ✓
(2)</td><td>✓ using $\tan\alpha$
✓answer</td></tr>
<tr><td>4.2.2</td><td>$Q\hat{P}R = 180° - (x+y)$ and
$\sin\ Q\hat{P}R = \sin[180° - (x+y)]$
$= \sin(x+y)$
In Δ PQR, by the sine rule
$\frac{PR}{\sin \hat{Q}} = \frac{QR}{\sin Q\hat{P}R}$
$\frac{PR}{\sin y} = \frac{15}{\sin[180° - (x+y)]}$
$\therefore PR = \frac{15 \sin y}{\sin(x+y)}$
(4)</td><td>✓ $R\hat{P}Q = 180° - (x+y)$

✓ choice of sine formula

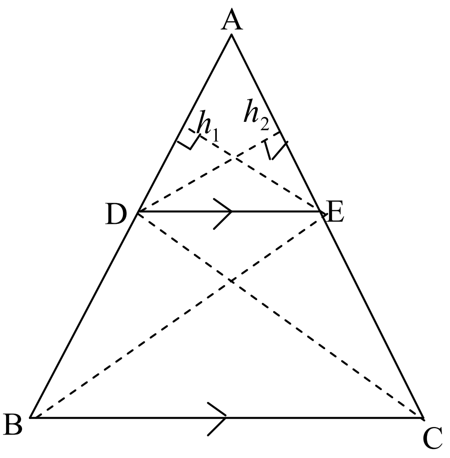
✓ correct substitution into sine formula
✓ $\sin(x+y)$</td></tr>
</table>

4.2.3	PW = PR tan α from 3.3.1 PW = $\dfrac{15\ \sin\ y}{\sin\ (x+y)}.\tan\alpha$ = $\dfrac{15\ \sin\ y}{\sin\ (y\ +\ y)}.\tan\alpha$ = $\dfrac{15\ \sin\ y}{\sin 2y}.\tan\alpha$ = $\dfrac{15\sin y}{2\sin y.\cos y}.\tan\alpha$ = $\dfrac{7{,}5}{\cos\ y}.\tan\alpha$ $\therefore$ PW = $7{,}5\ \dfrac{\tan\alpha}{\cos\ y}$ (3) **[14]**	✓ PW = $\dfrac{15\ \sin\ y}{\sin\ (x+y)}.\tan\alpha$ ✓ simplification ✓ $\sin 2y = 2\sin y.\cos y$

QUESTION 5

5.1	Supplementary✓ (1)	Answer
5.2.1	It is the angle between tangent and radius. ✓ (1)	Answer
5.2.2	$\hat{S}_3 + \hat{S}_4 = 90^\circ$ ... Tan ⊥ radius $\hat{N}_1 + \hat{N}_2 = 90^\circ$ ✓**…** Tan ⊥ radius ✓ $\hat{S}_3 + \hat{S}_4 + \hat{N}_1 + \hat{N}_2 = 90^\circ + 90^\circ$ ✓ $= 180^\circ$ $\therefore$ RNOS is a cyclic quadrilateral … opp ∠ quad supplementary ✓ (4)	Statement Reason Statement Reason
5.2.3	$\hat{S}_1 = x$ $S_1 = \hat{N}_2 = x$ ✓… Tan chord theorem✓ $\hat{N}_2 = S_3 = x$ ✓… base ∠'*s* of isosceles Δ OSN✓ $\hat{S}_3 = \hat{R}_2 = x$ ✓… ∠'*s* in same segment✓ $\hat{N}_2 = \hat{R}_1 = x$✓... ∠'s in same segment. ✓ (8)	Statement Reason Statement Reason Statement Reason Statement Reason
5.2.4	$\hat{O}_1 + \hat{Q}_2 + \hat{N}_2 + \hat{S}_3 = 180^\circ$ … sum of ∠'s of Δ OSN✓ But $\hat{S}_3 = \hat{N}_2$ … ∠*s* opp. equal sides✓ $\therefore \hat{O}_1 + \hat{O}_2 + 2\hat{S}_3 = 180^\circ$ $\hat{O}_1 + \hat{O}_2 + O_3 = 180^\circ$ … ∠'s on a straight line ✓ $\therefore \hat{O}_1 + \hat{O}_2 + 2\hat{S}_3 = \hat{O}_1 + \hat{O}_2 + \hat{O}_3$ $2\hat{S}_3 = \hat{O}_3$ ✓ $\therefore \hat{S}_3 = \frac{1}{2}\hat{O}_3$ (4) **OR** $\hat{O}_3 = S\hat{R}N$ ✓....ext.∠s of cyclic quad. ✓ $\hat{R}_2 = \hat{S}_3$ … ∠s in the same segment ✓ but SO = ON … radii of a circle $\therefore$ $\hat{R}_1 = \hat{R}_2$ ...= chords ; = ∠s ✓ $\therefore$ $\hat{S}_3 = \frac{1}{2}\hat{O}_3$ (4) **[18]**	Statement with reason Statement with reason Statement with reason $2\hat{S}_3 = \hat{O}_3$ Statement Reason Statement with reason Statement with reason

QUESTION 6

6.1	Given: ΔABC with D on AB and E on AC such that DE//BC  <u>RTP:</u> $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ Construction: Join DC and BE✓ Proof: $\dfrac{\text{area } \Delta \text{ ADE}}{\text{area BDE}} = \dfrac{\frac{1}{2}AD h_1}{\frac{1}{2}DB.h_2} = \dfrac{AD}{DB}$ ✓… areas of Δ's with the same height and common vertex are in the same ratio as their bases✓ $\dfrac{\text{area } \Delta \text{ ADE}}{\text{area DEC}} = \dfrac{\frac{1}{2}AE.h_2}{\frac{1}{2}EC.h_2} = \dfrac{AE}{EC}$ ✓ but area Δ DBE = area Δ DEC ✓... Same base, same parallel lines✓ Thus $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (6)	construction Statement Reason NOTE: If area of Δ found, then reason not necessary Statement Statement Reason
6.2.1	In ΔPQM, GH ∥ QT $\dfrac{QH}{HM} = \dfrac{GP}{GM}$ ✓(line parallel to one side ✓of a Δ **OR** Prop Th: GH ∥ PQ) $= \dfrac{1}{2}$ ✓ (3)	Statement Reason Answer
6.2.2	QH = k ; HM = $2k$ $\therefore$ RM = $3k$ ✓ MR = QM = $3k$ ...M is the midpoint of QR $\dfrac{RG}{RT} = \dfrac{RH}{RQ}$ ✓ (line parallel to one side of a Δ **OR** Prop Th: ✓GH ∥ PQ)) $= \dfrac{5k}{6k}$ ✓ $= \dfrac{5}{6}$ ✓ (5)	RM = $3k$ Statement Reason Substitution answer

6.3.1	Let $\hat{Z}_2 = x = \alpha$ ✓…. Tan chord theorem ✓ Then $A\hat{B}X = 90^o - \alpha$ … sum of $\angle's$ of Δ ABP ✓ But $\hat{Z}_1 = A\hat{B}P$ $= 90 - \alpha$ ... $\angle's$ opposite equal sides:AZ = AB ✓ $\hat{Z}_1 + Z_2 = \alpha + 90^o - \alpha$ … adj. $\angle's$ on a straight line ✓ $= 90^o$ Thus $\hat{Z}_3 = 90^o$ (5)	Statement Reason Statement with reason Statement with reason Statement with reason
6.3.2	In Δ AYZ and Δ AZX 1. $\hat{Z}_2 = \hat{X}$ … Tan chord theorem ✓ 2. $\hat{A}_2 = \hat{A}_2$ ….common ✓ 3. $A\hat{Y}Z = A\hat{Z}X$ (remaining angles) $\therefore \Delta$ AYZ /// Δ AZX $\angle,\angle,\angle$ ✓ (3)	Statement with reason Statement Statement with reason
6.3.3	$\therefore \frac{AZ}{AY} = \frac{AX}{AZ}$ Δs ||| sides in proportion ✓ $\therefore AZ^2 = AY \cdot AX$ (1) **[23]**	statement

<u>TOTAL MARKS</u>: [125]